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ROYAL SIGNALS & RADAR ESTABLISHMENT

MODELS FOR SURFACE SCATTERING FROM SOLIDS AND LIQUIDS

Author: D L Jordan

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SUMMARY

reported

The aim of this memorandum is to outline some work, on surface scattering of electromagnetic radiation. ~~In particular it is intended to present some of the~~ data ~~that~~ suggests that fractals seem to provide realistic models for rigid surface scattering. For agitated fluid surfaces the experimental data is more tenuous. Some new but limited experimental findings ~~that are reported here, together with other briefly mentioned work~~ suggests that something approximating to a marginal fractal may be more appropriate for liquids. Keywords: Fractals, Electromagnetic radiation, (RM)

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MODELS FOR SURFACE SCATTERING FROM SOLIDS AND LIQUIDS

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CONTENTS

1 INTRODUCTION

Fluctuations in the intensity of waves scattered by random media are of interest both from their importance as a source of information and in a noise context(1). The simplest mathematical model known to generate scintillation effects is the phase changing screen. This introduces spatially random distortions into an incident planar wavefront; amplitude fluctuations then develop during the course of free propagation beyond the scattering plane. This model, which may be used to represent scattering by an ideal rough surface has been extensively investigated for the case when the surface-height fluctuations constitute a joint Gaussian process that is then completed by specification of the height-fluctuation spectrum.

Two extreme surface types have been recognised for some time(2). The one that has received the most attention because of its simplicity and mathematical tractability is the smoothly varying surface which is differentiable to all orders and has Gaussian-like spectral properties(4). The other extreme type is the fractal model which assumes that the surface is continuous but not differentiable and posesses a power-law spectrum(4). An intermediate version called the sub-fractal model has also received a limited amount of attention; this assumes the surface height is continuous and differentiable but that its slope is a fractal. Unlike the situation for a fractal model, the concept of rays is valid but in the absence of higher surface derivatives no geometrical catastrophies occur in the propagating wave field as they do with smoothly varying models. Sub-fractal scatterers may be thought of as possessing a facet-like structure and have been investigated in connection with the propagation of radio waves through the ionisphere(5,6) and are also known to provide a model for the effect of internal waves on acoustic propagation in the oceans.

For a one-dimensional (corrugated) fractal surface the spatial power spectrum of the height fluctuations $P_h(k)$ varies as $|k|^{-(\nu+1)}$ where the fractal index ν is in the range $0 < \nu < 2$ and is related to the Hausdorff-Besicovitch dimension D by $\nu = 2(2 - D)$. For a two-dimensional surface it varies as $|k|^{-(\nu+2)}$. Jordan et al(11) have reported detailed surface profile measurements of a fractal phase screen having $\nu = 1.05$, which is close to the analytically convenient Brownian fractal value of 1.0. When $\nu = 2$ the surface is described as a marginal fractal. The spatial power spectrum for a sub-fractal surface has the same form as that for a fractal but with $2 < \nu < 4$. There is therefore a continuous evolution from an extremely ragged looking surface when ν is very small to a relatively smooth looking one when ν approaches its upper limit.

The aim of this memorandum is to suggest that whereas fractal surfaces with $\nu \sim 1$ seem to provide a useful model for rigid surface scattering over a significant range of scale sizes, liquid surfaces may be more appropriately described by marginal or sub-fractal models with $\nu \sim 2$. These suggestions, particularly for liquids, are extremely tentative; they are based on very limited experimental data. Indeed, it is possible that a model in which small scale structure comparable in size to the probing electromagnetic radiation wavelength and superimposed on very large scale structure is more appropriate in describing radar scattering from the sea. This so-called composite model does not possess a power law spectrum of the type appropriate to fractal or sub-fractal scatterers; it has power concentrated in two spectral regions. Although it has had some success in predicting the results of high resolution radar scattering from the sea it fails to correctly predict some polarisation phenomena and will not be discussed further in this Memorandum.

2 RIGID-SURFACE SCATTERING

Sayles and Thomas[8] showed that a whole range of solid surfaces covering a total range of scale sizes from 10^{-6} m to 10^{2} m could be described as fractal with a fractal dimension D between 1 and 2. Jordan[9] used Ordnance Survey data to show that ground terrain in Britain could also be realistically described as fractal in nature over scale sizes from some tens of metres to several kilometres and with a fractal index ν ranging between about 1.4 and 1.7. Similar conclusions were reached by Mark and Aronson[10] about terrain in America. We have also studied a range of solid surfaces over scale sizes appropriate to CO_2 (λ = 10.6 μm) laser scattering. In each case the surface profile was measured using a 'Talysurf' stylus instrument and the structure function and power spectrum determined[11]. Surfaces that have been studied include sandblasted metals, roughened dielectrics, sandpaper, brick, MOCVD grown GaAs and perspex that was heated and allowed to cool (to exhibit plastic flow). In all cases they behaved as fractals with a fractal index in the range $1 < \nu < 1.5$ over a range of scale sizes spanning one or two decades; results were always limited at small scale sizes by measurement limitations.

3 LIQUID-SURFACE SCATTERING

High resolution micowave radar returns from the sea have been analysed by both Jakeman and Pusey[13] and Ward[14] and shown to follow a K-distribution under many conditions. The experiments of Jordan et al[15] indicate that this is not generally characteristic of fractal scattering. Jakeman[12] however has shown analytically that scattering from a Brownian sub-fractal does lead to K-distributed intensity fluctuations. The internationally agreed ocean wave power spectrum (Phillips Law) follows an approximate $|k|^{-4}$ dependence (at least for short wind fetches) which is characteristic of a two-dimensional sub-fractal or marginal fractal. Evidence in support of a marginal fractal as being an appropriate model for sea surfaces has also been given by Glazman and Weichman[16]. Consequently it appears at least plausible that a sub-fractal (or if ν is exactly equal to two, a maginal fractal) may be a useful model for the evaluation of at least some types of sea scattering. Conceptually it also seems plausible because a sub-fractal, being an integrated fractal, should possess a relatively smooth surface profile. To illustrate this Fig 1(a) shows a 'Tallysurf' trace of a germanium fractal scatterer having a fractal index ν = 1.05, the slope of the power spectrum being approximately -2[11]. Fig 1(b) shows the integrated version of this fractal, the profile being integrated about its mean value. The relatively smooth shape, which would be expected for small non-breaking waves on a liquid is readily apparent.

4 EXPERIMENT

In an attempt to test the hypothesis that water waves may be modelled as sub-fractals, some limited small scale preliminary experiments were carried out. Waves were generated by an unbalanced electric motor stirring a paddle in a small glass tank (350 x 220 x 230 mm) containing water, and photographed through the side. The wave

height versus distance along the tank was then measured from these photographs and the surface height spatial power spectrum calculated. The experiments were very limited in dynamic range; to study phenomena which decays rapidly with spatial wavevector k requires large waves and high resolution. The waves produced in the tank were only about 15 mm high and the resolution was of order 0.2 mm. Six different motor speeds and positions were tried and their resultant spatial power spectra evaluated. All exhibited spectra with gradients between −2.7 and −3. A gradient of −3 is characteristic of a one-dimensional sub-fractal scatterer. A typical spectra is shown in Fig 2; it flattens-off at higher spatial frequencies because of measurement limitations.

Additional support for the possibility of waves on liquids being modelled as some form of fractal comes from measurement of the contrast of 10.6 μm CO_2 laser radiation scattered from waves produced in the small tank experiment. Chopped c/w laser radiation was focussed onto the water surface at an angle of approximately 60° to the mean surface normal by a 250 mm focal length lens, the arrangement satisfying the Fresnel condition $kKW^2 >> 1$ where

$$K = \frac{1}{2}\left[\frac{1}{\sigma} + \frac{1}{R}\right]$$

σ is the radius of curvature and W the width of the Gaussian profiled beam incident on the scatterer; R is the water to detector distance (100 mm). By varying the lens to water separation the wavefront curvature at the water surface and hence the effective water-detector distance could be varied; the maximum illuminated spot size on the water was ~2 mm. The water was agitated at 3-5 Hz. The scattered radiation was detected by a 100 μm pyroelectric detector connected to a phase sensitive detector; the output from this was fed into a digital voltmeter and thence into a computer. The contrast C of the scattered intensity is defined as

$$C^2 = \frac{\langle I^2 \rangle}{\langle I \rangle^2} - 1$$

and was determined by taking approximately 1000 measurements at each position of the focussing lens.

Figure 3 shows the variation of contrast in the Fresnel region as the wave tank to focus distance was varied. It can be seen that the contrast rises rapidly to ~1.7, with no sign of a fall-off at larger values of the target-focus distance. These results can be compared with theoretical curves derived by Jakeman(12) for Fresnel region scattering by a corrugated random surface with fractal slope; (a one-dimensional sub-fractal). Care must however be taken in interpreting these results as Jakemans theory assumes an infinitely wide illuminated area, thereby admitting contributions at the detector from as many water surface 'facets' as the effective water-detector distance will allow. In this experiment however only a relatively small illuminated area (< 2 mm) was used. The non-normal incidence of the laser beam will also slightly affect the situation. Nevertheless, in spite of these limitations it is expected that the theory should be approximately valid for relatively small water-focus distances. For larger separations the theory will be less applicable.

Jakemans results are given in terms of plots of C^2 versus $\beta(= kz^3/4L^2)$ for various outer scale sizes ξ and strength parameters α; the curve appropriate to the experimental conditions reported in this memorandum is shown in Fig 3. In evaluating this z is the effective water-surface to detector distance and L is the length over which the rms slope difference is unity. Measurement of the wave profiles allows a determination of the slope structure function $S(\delta)$ to be made. L was determined as ~ 4 x 10^{-2}m. An estimate of the outer scale size ξ suggests that it is of order 100 mm, resulting in a value of the

strength parameter α (= $k^2\xi^3/2L)^{1/6}$ of ~ 40. As can be seen from Fig 3 the experimental results are in general agreement with the theoretical ones; the agreement at large distances from the focus are possibly fortuitous because of the theory limitations referred to earlier. The large value of α in these experiments ensures that the contrast would not be expected to fall until very large target-focus distances were reached. The surprisingly good agreement between theory and experiment lends support to the idea that at least some types of disturbed liquid surfaces may be modelled as a fractal-type surface with $\nu \sim 2$.

5 CONCLUSIONS

In this memorandum we have briefly outlined some of the evidence that suggests that over a significant range of scale sizes rigid surfaces can be interpreted as being fractal in nature (with upper and lower scale cut-offs). A limited amount of evidence, both circumstantial and tentative experimental in nature has also been presented that suggests that at least some types of disturbed liquid surfaces can also be described as exhibiting a fractal-like surface, but with higher values of the fractal index ν. The wave profile data produced in these experiments indicates that ν was in the range 1.7 to 2. The contrast measurements were surprisingly well fitted by a sub-fractal (ν = 2) model. These tentative findings, together with the form of the internationally agreed ocean spectra (Phillips Spectrum) and the work of Glaseman and Weichman(16) on the relevance of marginal fractals in describing sea surfaces, suggests that a fractal-type model with $\nu \sim 2$ is at least a reasonable first step. Whether it should be referred to as a fractal, marginal fractal or sub-fractal surface is at present unanswerable, and to a large extent academic.

Considerably more work is urgently needed in this important area. Any future experimental results should not only be compared with predictions based on the models discussed here but also with the composite model discussed in the introduction. To this end a programme of work on visible and infra-red grazing angle scattering in both forward and backscattering configurations is planned. It will attempt, amongst other things, to simulate radar measurements of the ratio of vertically to horizontally polarised radiation from various surface types and hopefully to elucidate the scattering mechanisms.

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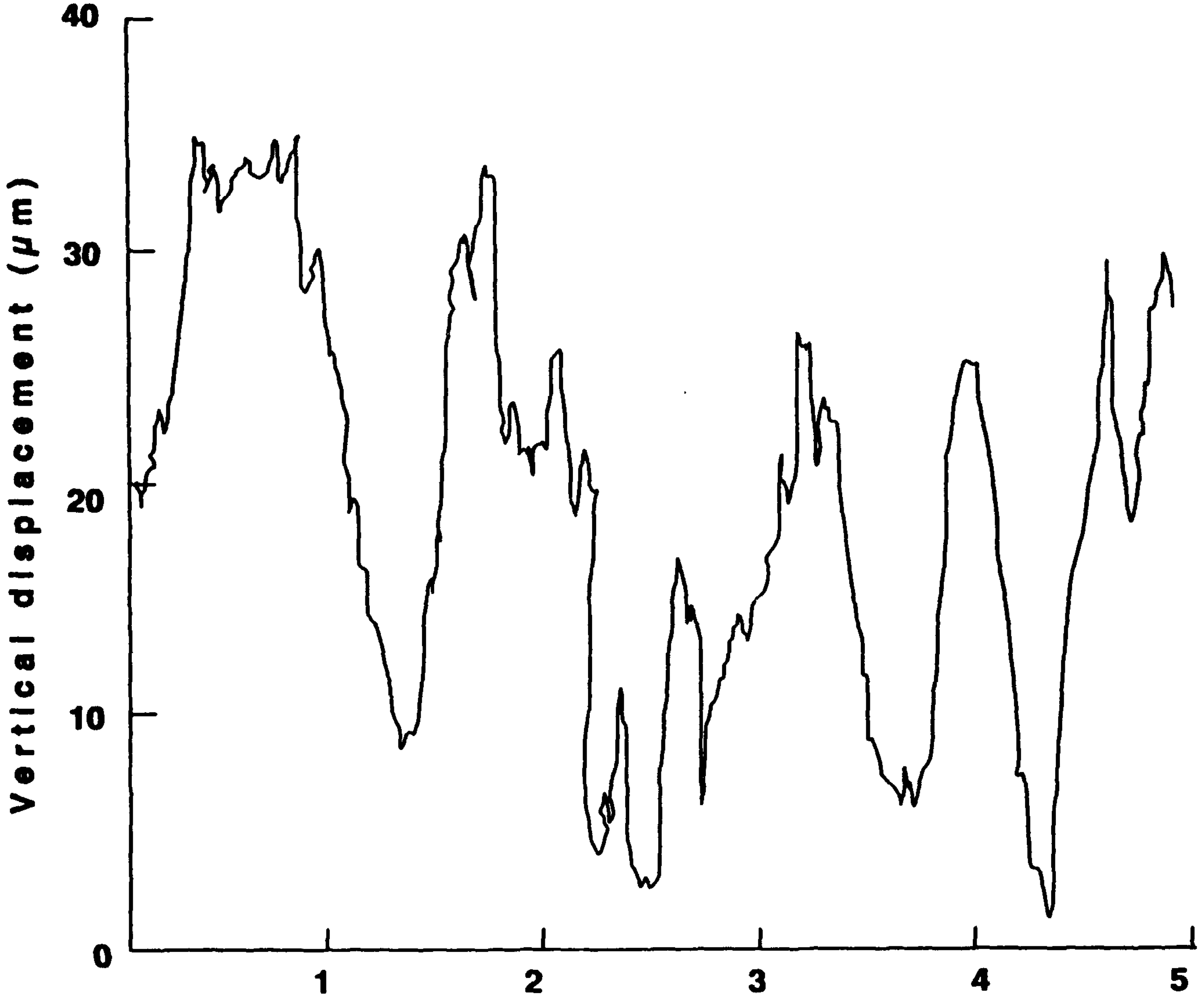

FIGURE 1(a). TALLYSURF PROFILE OF ROUGHENED GERMANIUM SURFACE

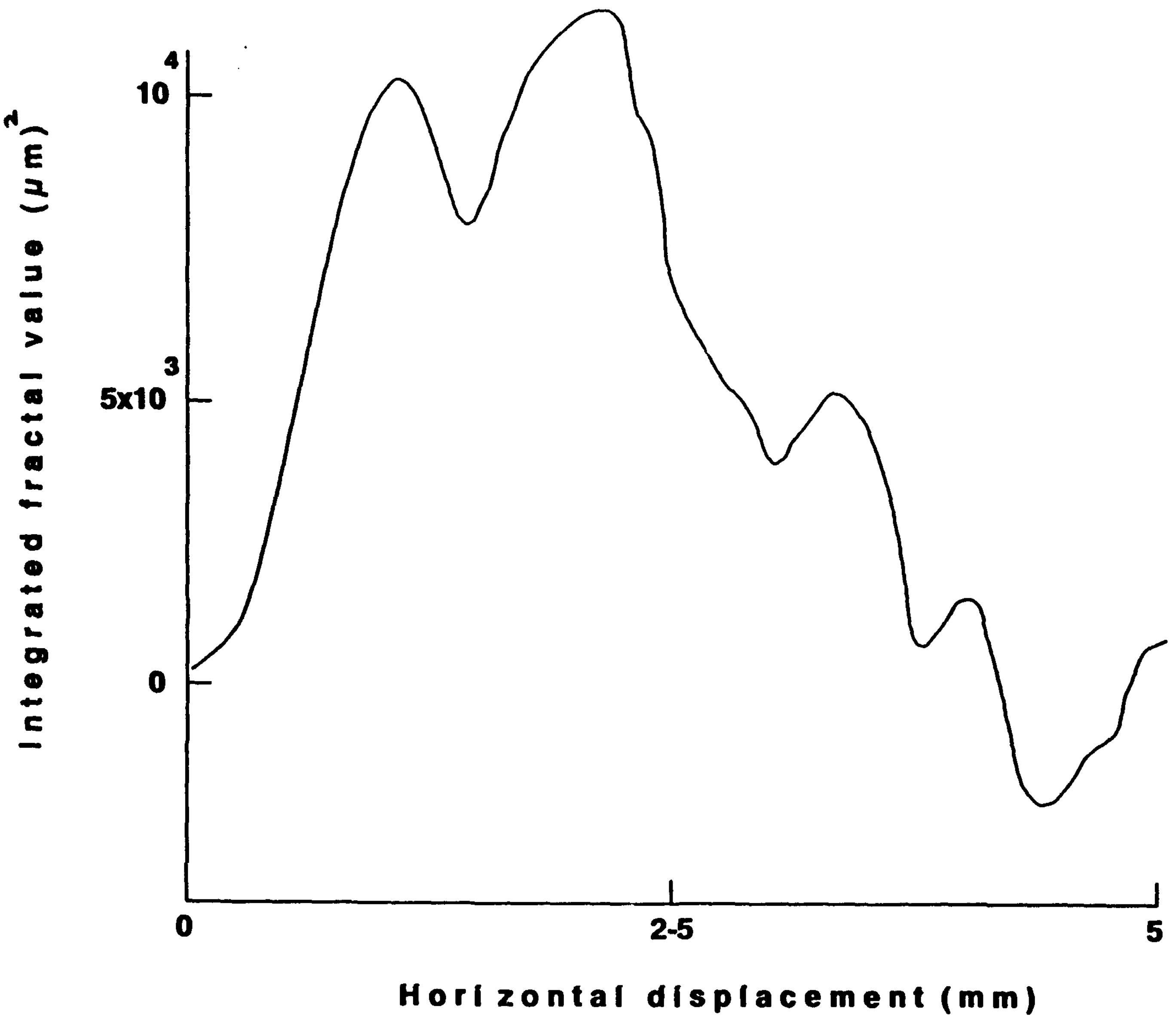

FIGURE 1(b). INTEGRATED VERSION OF FIGURE 1(a)

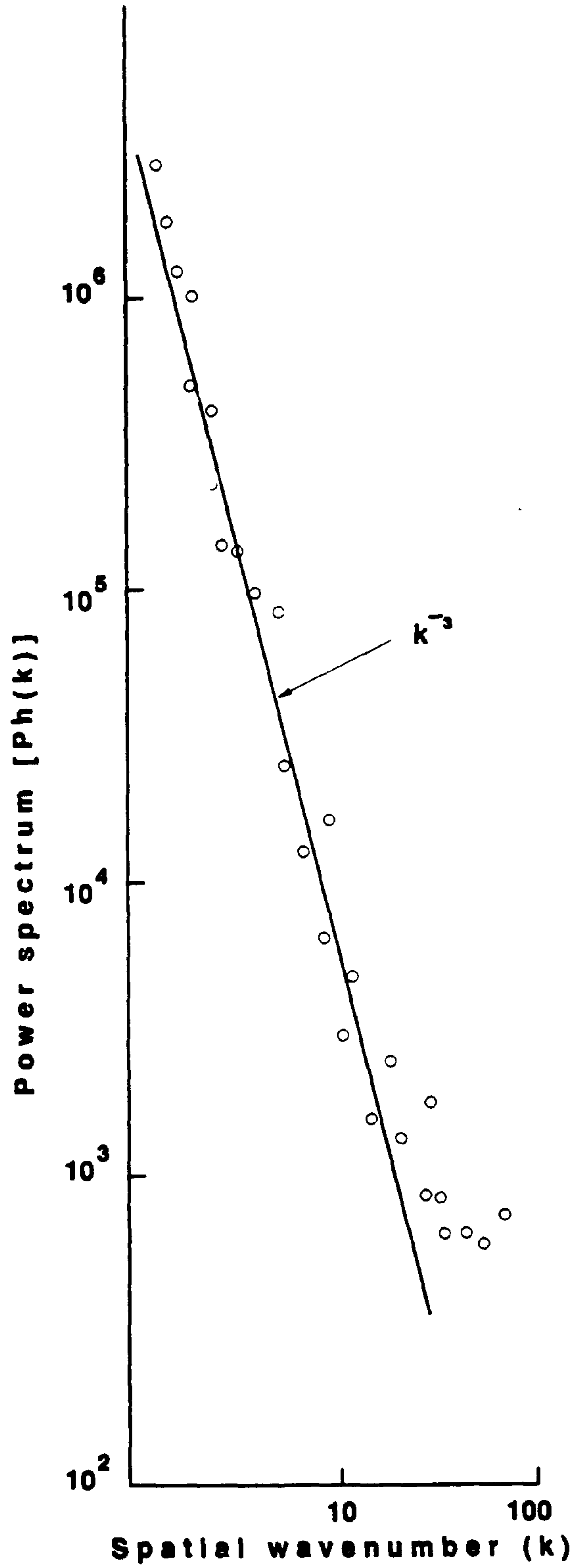

FIGURE 2. POWER SPECTRUM (ARBITRARY UNITS) OF WATER

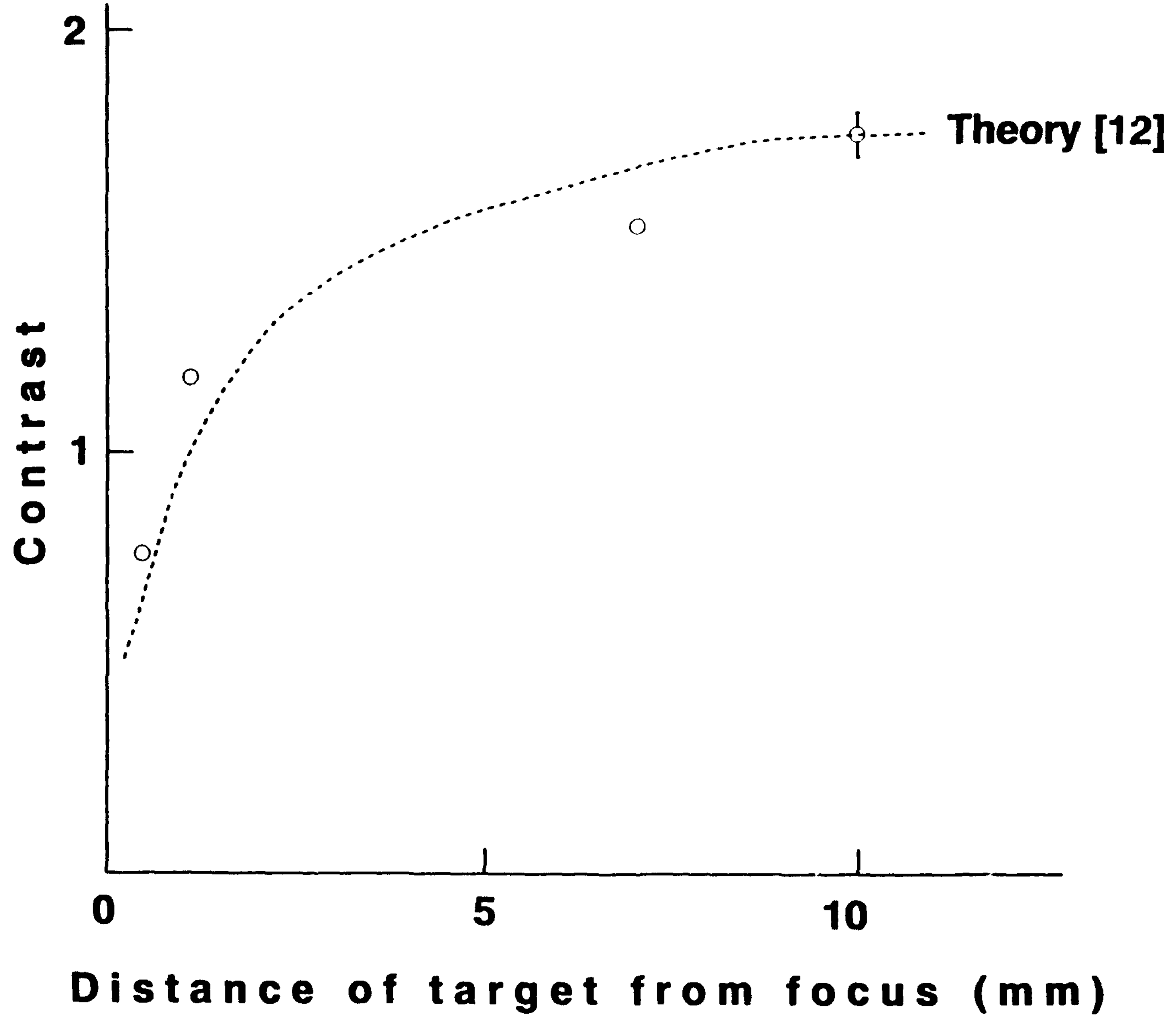

FIGURE 3. CONTRAST OF SCATTERED RADIATION AS A FUNCTION OF DISTANCE OF MEAN WATER SURFACE FROM FOCUS

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Abstract

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